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THEORY OF AN INDUCTION MHD PROPELLER WITH A FREE FIELD

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A considerable number of papers have been published [1-6] on magnetohydrodynamic propellers. However, the successes achieved in recent years in creating and using in technology superconducting magnetic systems [7] are an impetus for further investigations into MHD propellers.

This paper is devoted to an investigation of the energy characteristics of a so-called induction MHD system with a free field [2]. This work is made necessary by the fact that in [2] the energy characteristics of the MHD propellers under consideration were obtained without taking into account the longitudinal boundary effect and are therefore grossly overstated. Subsequently the results in [2], without critical analysis, were reproduced in other publications [3, 6] devoted to MHD propellers.

The investigation carried out in this work showed that taking account of the finiteness of the dimensions of the source of the electromagnetic field leads not only to quantitative changes. At the same time the effectiveness of the installation for a given magnetic field intensity is substantially below the predictions [2], the required magnetic fields for obtaining a given efficiency are considerably higher. In the paper we propose a method for increasing the effectiveness of the induction MHD propeller under consideration as a result of "amplitude modulation"; in this case the energy characteristics of the propeller (of finite dimensions) can be to a certain degree brought nearer to an "ideal" propeller [2].

1. We consider a rigid body of finite dimensions located in boundless conducting liquid with conductivity σ, density ρ, being brought in motion by electromagnetic forces; the source of the fields is located within the body. In the role of the rigid body we consider the simplest model – a flat plate of finite width $2a$ along the x axis, infinitely extending along the z axis, moving in its plane in the direction of the negative x half-axis. The assumption about infinity along the z axis is of no major importance. The results obtained will be true if the long plate being considered is rolled into a "ring" or cylinder with a height of $2a$ and a radius substantially exceeding the wavelength $2\pi/k_1$.

The source of the electromagnetic field in the surrounding liquid is provided by introduction, in the plane of the plate, of surface currents having z-direction and being distributed over the width of the plate:

$$i_z(x_1, t) = \mathrm{Real}\, J_0 \cdot i_0(x_1)\, e^{i(k_1 x_1 - \omega_0 t)} \quad (|x_1| \leqslant a) \tag{1.1}$$

(these currents act in the role of an inductor). In (1.1) J_0 is the maximum current density, the function $i_0(x_1)$ characterizes the distribution of the current amplitude over the plate width, $|i_0(x_1)| \leqslant 1$. By x_1, y_1 here and below we denote the coordinates with dimensions; for the corresponding dimensionless quantities we use the symbols x, y without indices. The problem consists of determining the distribution of the fields E, H of total force acting on the plate with the currents (1.1) from the side of the magnetic field of the currents j in the liquid, the required electric power, and also the velocity u_0 which is acquired by the plate.

Below it is shown that within fairly wide limits of parameters the assumption about smallness of the parameter of magnetohydrodynamic interaction is valid:

$$N = \frac{\sigma H_0^2 2a}{\rho c^2 u_0} \ll 1. \tag{1.2}$$

The problem now becomes simpler. In particular, the electromagnetic fields in the liquid are determined from the equations

$$\mathrm{rot}\,\mathbf{H} = (4\pi/c)\,\mathbf{j}, \quad \mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \quad \mathrm{div}\,\mathbf{H} = 0, \\ \mathbf{j} = \sigma[\mathbf{E} + (1/c)\,\mathbf{v}\times\mathbf{H}], \tag{1.3}$$

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where $\mathbf{v}$ is a known field of velocities which is obtained in the case of flow around the given body without the electromagnetic forces taken into account. In the case of a flat plate this field of velocities can be taken in the form $\mathbf{v} = u_0 \mathbf{e}_x$ which does not take into account the presence of a boundary layer. The latter is valid if the space close to the plate occupied by the fields E, H is substantially larger than the volume occupied by the boundary layer, i.e., in the case

$$\lambda = 2\pi/k_1 \gg \delta, \tag{1.4}$$

where δ is the thickness of the boundary layer.

2. The fields **E**, **H**, satisfying Eqs. (1.3), are sought in terms of the vector potential

$$\mathbf{A} = J_0 (2a/c) A(x, y) e^{-i\omega_0 t} \mathbf{e}_z, \tag{2.1}$$

where in terms of the dimensionless variables x, y, introduced by means of the length scale $2a$, the dimensionless complex function A(x, y) satisfies the following equation and boundary conditions:

$$\Delta A(x, y) + \left[ik_0 A(x, y) - s\frac{\partial A(x, y)}{\partial x}\right] \mathrm{Re}_m = 0; \tag{2.2}$$

$$\left.\frac{\partial A}{\partial y}\right|_{y=0} = -2\pi i_1(x) e^{ik_0 x}, \quad i_1(x) = \begin{cases} i_0(x) & \text{for} \quad |x| \leqslant 1/2, \\ 0 & \text{for} \quad |x| > 1/2; \end{cases} \tag{2.3}$$

$$A|_{y=\infty} = 0. \tag{2.4}$$

The dimensionless magnetic number of Reynolds Re_m and the slip s entering into (2.2)

$$\mathrm{Re}_m = 4\pi\sigma v_f^0\, 2a/c^2, \quad s = u_0/v_f^0 \tag{2.5}$$

are defined in terms of the phase velocity of the running wave

$$v_f^0 = \omega_0/k_1 \tag{2.6}$$

and the velocity of the liquid u_0 (v_f^0 and u_0 are defined in the plate system). In the following we require also the magnetic Reynolds number determined with respect to the velocity of liquid u_0

$$\mathrm{Re}_m^0 = 4\pi\sigma u_0 2a/c^2 = \mathrm{Re}_m s. \tag{2.7}$$

The parameter

$$k_0 = k_1 2a = n\pi, \; n = 2a/(\lambda/2) \tag{2.8}$$

characterizes the number of half-waves $\lambda/2$ of the current (1.1) covering the width of the plate. The boundary conditions (2.3) and (2.4) have been written for the upper half-space $y \geqslant 0$. We confine ourselves to the consideration of the region $y \geqslant 0$ because of symmetry of A(x, y) about the plane y = 0.

The solution of the problem (2.2)-(2.4) is constructed by means of the Fourier transform. Having represented

$$i_1(x) = \int_{-\infty}^{\infty} I(k) e^{ikx} dk, \quad I(k) = (1/2\pi)\int_{-\infty}^{\infty} i_1(x) e^{-ikx} dx = (1/2\pi)\int_{-1/2}^{1/2} i_0(x) e^{-ikx} dx,$$

the solution can be obtained in the form

$$A(x, y) = 2\pi \int_{-\infty}^{\infty} \frac{I(k - k_0) e^{ikx} e^{-\sqrt{k^2 - i\mathrm{Re}_m(k_0 - ks)}\, y}}{\sqrt{k^2 - i\mathrm{Re}_m(k_0 - ks)}} dk, \tag{2.9}$$

where I(k) is the spectral density of the step function $i_1(x)$ (2.3); $I(k - k_0)$ is the spectral density of the function $i_1(x) e^{ik_0 x}$; under $\sqrt{k^2 - i\mathrm{Re}_m(k_0 - ks)}$ we understand the value of the root with a positive real part.

It should be noted that no constraints are used on the value of the magnetic Reynolds number in (2.9).

From (2.1), (2.9) it is seen that the electromagnetic field in the liquid constitutes a superposition of fields running in the x direction with a fixed frequency ω_0 and phase velocities varying from $-\infty$ to ∞, since the phase velocity

$$v_f = (k_0/k) v_f^0, \tag{2.10}$$

where v_f^0, k_0 are given in (2.6), (2.8), corresponds to the wave corresponding to the dimensionless wave number k.

3. We calculate the integral quantities (the force of pull acting on the plate and the required electrical power) referred to unit length of the plate along the z axis. The force acting on the plate with the currents (1.1) from the side of the magnetic field has only an x-component which is computed in the form

$$F_x = -(1/c)\int\limits_{-a}^{a} i_z(x_1, t)\, H_y(x_1, 0, t)\, dx_1$$

or after going over to complex quantities and averaging over time

$$\langle F_x\rangle = \frac{2aJ_0^2}{2c^2}\,\text{Real}\int\limits_{-\infty}^{\infty} i_1(x)\, e^{ik_0x}\,\frac{\partial A^*}{\partial x}\Big|_{y=0} dx = -2a\,\frac{J_0^2\,(2\pi)^2}{2c^2}\,\text{Real}\int\limits_{-\infty}^{\infty}\frac{ik}{\sqrt{k^2 - i\,\text{Re}_m\,(k_0 - ks)}}\,|I(k-k_0)|^2\, dk.$$

Calculating the real part of the expression behind the integral sign, we can bring the result to the form

$$\langle F_x\rangle = -a\,(H_0^2/4\pi^2)\,F_1(k_0, \text{Re}_m, s),$$

$$F_1 = 4\pi^2\left[\int\limits_{-\infty}^{k_0/s} k\Phi(k)\,|I(k-k_0)|^2\, dk - \int\limits_{k_0/s}^{\infty} k\Phi(k)\,|I(k-k_0)|^2\, dk\right], \tag{3.1}$$

where under Φ(k) we understand the positive-definite function for all k

$$\Phi(k) = \frac{1}{\sqrt{2}}\,\frac{\sqrt{\sqrt{k^4 + \text{Re}_m^2\,(k_0 - ks)^2} - k^2}}{\sqrt{k^4 + \text{Re}_m^2\,(k_0 - ks)^2}}. \tag{3.2}$$

The electric power Q required, representing the sum of powers going on carrying out the mechanical work on the liquid and on its Joule heating, is computed as the flux of the Poynting vector S through the surface (in the plane problem being considered, through the contour) enveloping the plate, i.e.,

$$Q = 2\int\limits_{-a}^{a} S_y|_{y_1=0}\, dx_1, \quad S_y = (c/4\pi)\,E_z H_x, \quad \langle S_y\rangle = \frac{\omega_0}{8\pi}\,\frac{J_0^2 2a}{c^2}\,\text{Real}\left[iA(x, 0)\,\frac{\partial A^*}{\partial y}\Big|_{y=0}\right].$$

Since $S_y|_{y_1=0} = 0$ for $|x_1| > a$, the limits of integration in Q can be replaced by [−∞, ∞]; substituting the solution (2.9) into the expression $\langle S_y\rangle$, we can bring the sought quantity to the form

$$\langle Q\rangle = (\omega_0/8\pi^2)\,(2a)^2\,H_0^2 Q_1(k_0, \text{Re}_m, s),$$

$$Q_1 = 4\pi^2\left[\int\limits_{-\infty}^{k_0/s}\Phi(k)\,|I(k-k_0)|^2\, dk - \int\limits_{k_0/s}^{\infty}\Phi(k)\,|I(k-k_0)|^2\, dk\right]. \tag{3.3}$$

In (3.1), (3.3) by H_0 we understand the quantity

$$H_0 = 2\pi J_0/c,$$

having the meaning of maximum tension of the magnetic field. The efficiency of the model under consideration is determined in the form

$$\eta = -\frac{\langle F_x\rangle u_0}{\langle Q\rangle} = \frac{s}{k_0}\,\frac{F_1}{Q_1}.$$

From (3.1), (3.3) it is seen that the different portions of the spectrum $I(k - k_0)$ of the function $i_1(x)e^{ik_0x}$ introduce different contributions into the quantity $\langle F_x\rangle$, $\langle Q\rangle$. From this viewpoint the entire spectrum can be divided into three portions:

I ($k < 0$) – the contribution into F_1 is negative, into Q_1 it is positive;

II ($0 < k < k_0/s$) – the contribution into F_1 is positive, into Q_1 it is positive;

III ($k > k_0/s$) – the contribution into F_1 is negative, into Q_1 it is negative.

Hence it follows that the running magnetic fields only in the portion II work in the propeller regime. The portion III corresponds to the generator regime – here the liquid is braked by the field and performs work on the field, and the part of this work after subtraction of the Joule heating is transmitted into the electric system. The portion I corresponds to a heater – here the Joule heating exceeds the work performed by the liquid against the electromagnetic field. Consequently, not only the part of kinetic energy of the liquid being dissipated but also electric energy is transformed into heat.

These peculiarities have a physical explanation consisting of the fact that on the portion II of the spectrum the phase velocity of the constituent waves, in accordance with (2.10), (2.5), exceeds the velocity of the liquid u_0, i.e., $v_f^0 s = u_0 < v_f < \infty$, while on the portion III $0 < v_f < u_0$. On the portion I the waves run in the direction opposite to the flow of liquid.

4. The determination of the quantity H_0, necessary for ensuring the velocity u_0, is carried out from the equation of motion of the plate $\langle F_x \rangle + T_x = 0$, where $T_x = c_f 4a\rho u_0^2/2$ is the force of resistance of friction. Hence, with (3.1) taken into account, it follows

$$H_0 = 2\pi\sqrt{2\rho c_f/F_1}u_0. \tag{4.1}$$

We assume that the coefficient of resistance of the plate, being brought in motion by the electromagnetic method under consideration, is equal to the known coefficient of resistance [8] of a smooth plate

$$c_f = 0.455/(\lg \mathrm{Re})^{2.58} - 1700/\mathrm{Re},$$

found without the effect of electromagnetic fields taken into account, and being suitable in a broad range of Reynolds numbers up to $\mathrm{Re} \sim 10^9$. As a consequence of the fact that into (4.1) c_j enters in the form of the multiplier $\sqrt{c_f}$, the error incurred by the assumption made is not very large, and for the estimates of H_0 this error may be neglected.

Since c_f depends on Re, with

$$\mathrm{Re} = u_0 2a/\nu = \mathrm{Re}_m^0 \nu_m/\nu, \quad \nu_m/\nu = c^2/4\pi\sigma\nu$$

(for seawater $\sigma = 5 \cdot 10^{10}$ 1/sec, $\nu = 10^{-2}$ cm²/sec, $\nu_m/\nu = 1.43 \cdot 10^{11}$), the relation (4.1) can be represented in the form

$$H_0 = \mathrm{Re}_m^0 2\pi\sqrt{2c_f(\mathrm{Re})/F_1(k_0, \mathrm{Re}_m, s)} \cdot \nu_m\sqrt{\rho}/2a. \tag{4.2}$$

From (1.2), (4.1) for the parameter of MHD-interaction we have

$$N = \mathrm{Re}_m^0 2\pi c_f/F_1.$$

When Re varies from 10^7 to $3 \cdot 10^9$ (with $7 \cdot 10^{-5} < \mathrm{Re}_m^0 < 2.1 \cdot 10^{-2}$) the coefficient of resistance c_f varies within the limits $(3\text{-}1)10^{-3}$. Hence it is seen that there exist fairly wide possibilities for validity of the condition (1.2). Although it mentioned at once that for the example being considered below with large values $2a$, u_0 (and consequently, Re_m^0), regimes advantageous from the viewpoint of efficiency lead to values of F_1 for which $N \sim 0.25$. For smaller Re_m^0 fulfilment of the condition (1.2) is made easier.

5. To obtain qualitative representations about the dependence of F_1, Q_1 on k_0 in the case of a fixed Re_m we have to investigate the behavior of functions behind the integral sign in (3.1), (3.3). In Fig. 1 we have schematically represented the functions $\Phi(k)$ (3.2), $k\Phi(k)$, $|I(k-k_0)|^2$, and also $k\Phi(k)|I(k-k_0)|^2$, $\Phi(k)|I(k-k_0)|^2$ for a fixed k_0. The maximum value with respect to the modulus is acquired for the function $k\Phi(k)$ for $k_{1,2} = (\sqrt{3}/6)\left[-\mathrm{Re}_m s \pm \sqrt{\mathrm{Re}_m^2 s^2 + 4\sqrt{3}\,\mathrm{Re}_m k_0}\right]$; under the condition $\mathrm{Re}_m s \ll 1$, which is valid almost for all application with the use of seawater, $k_{1,2} = \pm\left(1/\sqrt[4]{3}\right)\sqrt{\mathrm{Re}_m k_0}$, with the maximum value of $|k\Phi(k)|$ being $\sqrt{2}/4$. In the case of $k^2 \gg \mathrm{Re}_m|k_0 - ks|$ the functions $\Phi(k)$, $k\Phi(k)$ are simplified; here $\Phi(k) = \mathrm{Re}_m|k_0 - ks|/2|k|^3$, and for $k = k_0$ $\Phi(k)$, $k\Phi(k)$ assumes the values $\mathrm{Re}_m(1-s)/2k_0^2$, $\mathrm{Re}_m(1-s)/2k_0$, respectively. An important part in the behavior of the integrals F_1, Q_1 is played by the function $|I(k-k_0)|^2$. Independently of the actual form of the step function $i_1(x)e^{ik_0x}$ in the power of the spectrum there exists a principal maximum at the point $k = k_0$ with width $\Delta k \sim 2\pi$ given by the indeterminacy relation. In addition to the principal maximum there exist secondary maxima whose intensity falls as we move away from the principal maximum; at the same time the law of decay of the secondary maxima is extremely important, since it is in fact determining the behavior of the functions behind the integral sign in the region of the origin of coordinates. As is seen from Fig. 1, the basic contribution into the integrals F_1, Q_1 is made by two segments of the k axis: $|k - k_0| \leqslant \Delta k$ and $|k| \leqslant \sqrt{\mathrm{Re}_m k_0}$. On the first segment (we call it the right segment) the functions behind the integral sign have the maxima $I^2(0)\,\mathrm{Re}_m(1-s)/2k_0$ and $I^2(0)\,\mathrm{Re}_m(1-s)/2k_0^2$, monotonically depending on k_0; the maxima of the functions behind the integral sign on the left segment, representing the secondary maxima reinforced by the multipliers $k\Phi(k)$ and $\Phi(k)$, of the function $|I(k-k_0)|^2$, nonmonotonically depend on k_0 (dependent on k_0, their position along k in the region $k = 0$ is displaced, and the values of the maxima under consideration vary as a result of rapid variation of the multipliers $k\Phi(k)$ and $\Phi(k)$). In Fig. 1 the functions $k\Phi(k)|I(k-k_0)|^2$, $\Phi(k)|I(k-k_0)|^2$ are presented for a value of k_0 for which the position of one of the secondary maxima coincides with the position of a maximum of the function $k\Phi(k)$.

Consequently, the dependence of $F_1(k_0)$, $Q_1(k_0)$ in the case of fixed Re_m, s has a nonmonotonic character up to certain critical values of k_0. The character of oscillations and the magnitude of the critical values of k_0 are determined by the function $|I(k-k_0)|^2$ or more precisely, by the law of decay of its secondary maxima.

The results for the case of a constant current amplitude across the width of the plate, i.e., for $i_0(x) \equiv 1$, $I(k) = (1/2\pi)\frac{\sin k/2}{k/2}$, are presented in Figs. 2-4. Here for k_0 varying within the limits from π to 15π, we have presented the

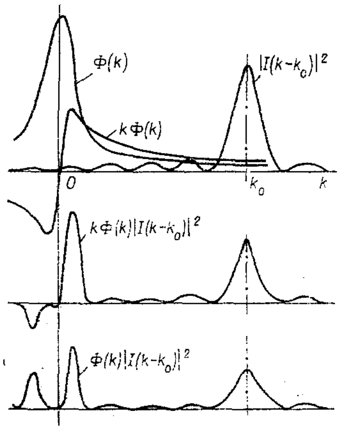

Fig. 1

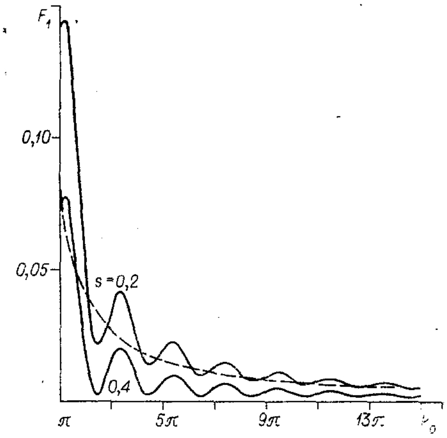

Fig. 2

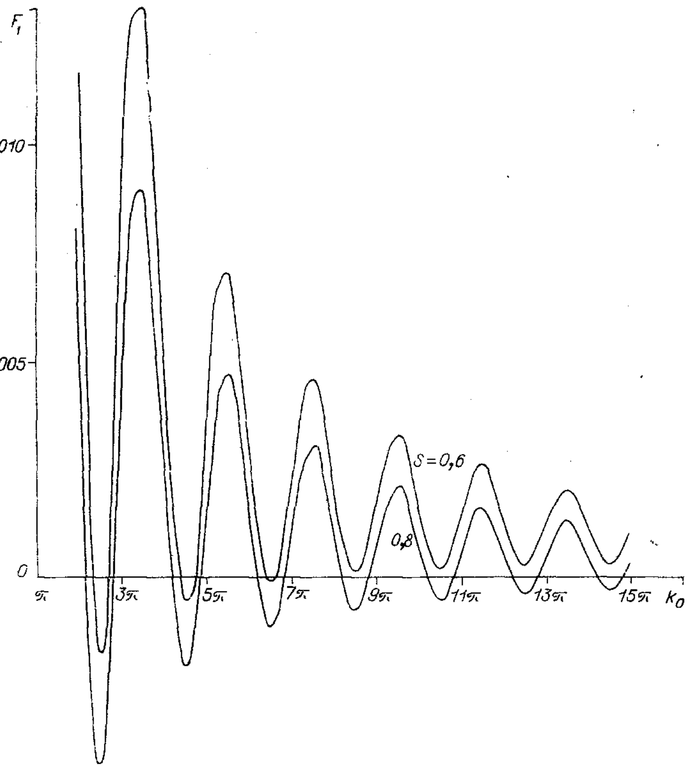

Fig. 3

relations $F_1(k_0)$ (Figs. 2, 3) and $\eta(k_0)$ (Fig. 4) for different values of s for $Re_m^0 = 0.02$. (To the value $Re_m^0 = 0.02$ there corresponds a body with length $2a = 200$ mm $= 2 \cdot 10^4$ cm moving at the velocity $u_0 = 1.43 \times 10^3$ cm/sec ≈ 50 km/h in seawater with conductivity $\sigma = 5 \cdot 10^{10}$ 1/sec; these parameters coincide with or are close to those considered in [2]). In Fig. 4 the relation $\eta(k_0)$ is presented fully only for s = 0.2, 0.4; for s = 0.6 the portions of curves going beyond the limits of the first two have been plotted. For s = 0.8 all local maxima of $\eta(k_0)$ have values that are less than those depicted on the graphs; therefore these relations have not been presented in Fig. 4.

In connection with the relations just presented we should pay attention to two features: first, efficiencies not attaining the magnitude 0.1 for any values of k_0, s are very far removed from the predictions of the theory [2] which does not take into account finiteness of the dimensions of the source of the electromagnetic field; second, F_1 and η not only oscillate about the mean positions when k_0 increases, but in the case of large s also go into the region of negative values. Although $v_f^0 > u_0$, on the plate instead of the force of pull there can act a braking force from the side of the

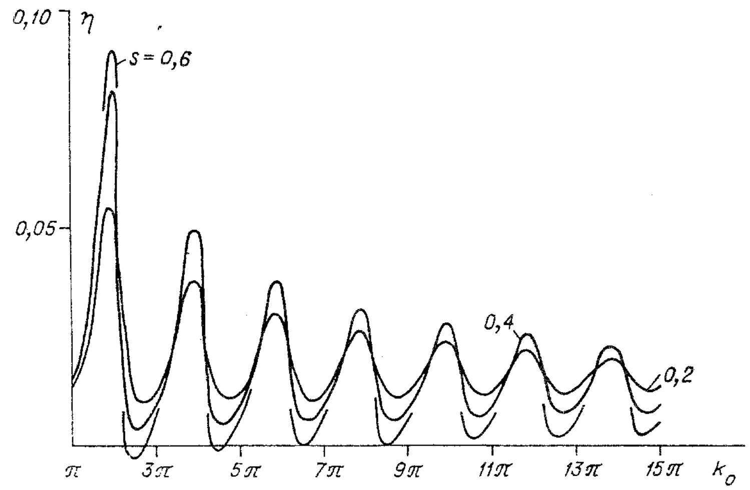

Fig. 4

magnetic field (on certain intervals of k_0). The occurrence of this unusual result is due to the portion III of the $I(k - k_0)$ spectrum.

From Figs. 2-4 it is seen that the oscillating character of the relations $F_1(k_0)$, $\eta(k_0)$ has a tendency to persist also beyond the limits $k_0 = 15\pi$. At the first glance it appears that this circumstance contradicts "common sense." Indeed, intuitively we imagine that if on the plate width $2a$ there are located "many" wavelengths, i.e., $k_0 \gg \pi$, and if $i_0(x) \equiv 1$ (the amplitude of current is constant across the width of the plate), then the plate of finite width being considered in a certain sense only slightly differs from an infinite plate. In fact it is assumed that the quantities $\langle F_x \rangle$, $\langle Q \rangle$, η coincide with the analogous quantities computed for a portion with width $2a$, imagined to be cut out of an infinite plate with the current $i_z(x_1, t) = J_0 e^{i(k_1 x_1 - \omega_0 t)} (|x_1| \leqslant \infty)$. The dimensionless quantities referring to the portion under consideration are denoted by F_∞, Q_∞, η_∞, where it can be shown that

$$F_\infty = 2\pi \operatorname{Re}_m (1 - s)/2k_0, \quad Q_\infty = 2\pi \operatorname{Re}_m (1 - s)/2k_0^2, \quad \eta_\infty = s. \tag{5.1}$$

The parameters Re_m, s, k_0 entering here have the previous meaning (2.5)-(2.7). In Fig. 2 for the value s = 0.2 the relation $F_\infty(k_0)$ is shown by the dashed line. We see that even in the region $k_0 = 15\pi$ the corresponding quantity F_1 still noticeably differs from F_∞. As for η, then within the limits $\pi \leqslant k_0 \leqslant 15\pi$ it is altogether very far from the quantity $\eta_\infty = s$.

Thus, if on the plate there is packed an order of 15 half-waves, such a plate is still far from an "infinite" plate. It is interesting to find out for which values k_0 the relations $F_1(k_0)$, $Q_1(k_0)$ from (3.1), (3,3) go to their asymptotic values (5.1) For an answer we turn to the functions F_1, Q_1 behind the integral sign, shown in Fig. 1. We note that exit to the asymptotic values occurs when the contribution of the left peaks of the functions $k\Phi(k)|I(k - k_0)|^2$ and $\Phi(k)|I(k - k_0)|^2$ behind the integral sign to the integrals becomes small in comparison with the contribution of the right peaks. Let us consider to what condition this circumstance leads with respect to F_1.

The contribution of the neighborhood of the point k = 0 to the integral F_1 can be small in two cases: first, if the value of the maximum of the function behind the integral sign in this region is substantially less than the corresponding maximum at the point $k = k_0$, i.e., if $1/k_0^2 \ll \operatorname{Re}_m \times (1 - s)/2k_0$, or

$$[(1 - s) \operatorname{Re}_m k_0]^{-1} \ll 1, \tag{5.2}$$

second, if on the segment $|k| \sim \sqrt{\operatorname{Re}_m k_0}$ there is concentrated a considerable number of secondary maxima of the function $|I(k - k_0)|^2$, i.e., $\sqrt{\operatorname{Re}_m k_0} \gg 2\pi$, or

$$2\pi/(\operatorname{Re}_m k_0) \ll 1 \tag{5.3}$$

(for this the contribution of the neighborhood k = 0 to the integral F_1 vanishes because of the function $k\Phi(k)$ being odd in this neighborhood). Since the values interest of the parameter s do not go beyond the limits $0.3 \leqslant s \leqslant 0.8$, the conditions (5.2), (5.3) practically coincide. As for the integral Q_1, here fulfilment of the condition (5.3) is not sufficient for Q_1 to go over to asymptotic behavior. In the case of (5.3) the contribution of the neighborhood k = 0 to the integral Q_1

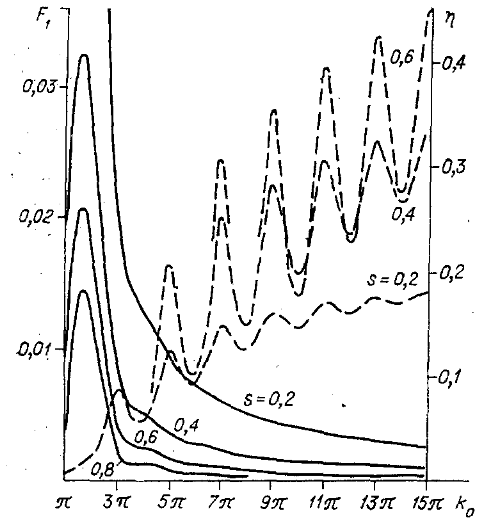

Fig. 5

does not vanish; it only ceases oscillating, dependent on k_0. The necessary condition is derived analogously to (5.2) and it assumes the form $4/(\sqrt{2\mathrm{Re}_m k_0} k_0^2) \ll \mathrm{Re}_m (1-s)/2k_0^2$ or

$$4[\sqrt{\mathrm{Re}_m k_0}\mathrm{Re}_m(1-s)]^{-1} \ll 1. \tag{5.4}$$

This is a stronger condition than (5.2) or (5.3), since, as was already mentioned, for almost all applications $\mathrm{Re}_m^0 \ll 1$. For example, for the case considered above with $\mathrm{Re}_m^0 = 0.02$ for s = 0.5, the condition (5.4) reduces to the requirement $k_0 \gg 2\cdot 10^6$. Consequently, the efficiency values $\eta \sim$ s of practical interest are attained for very large k_0 for which the quantity F_1, coinciding with F_∞ from (5.1), is small. This leads to prohibitively large values of the required field H_0, given by (4.1) or (4.2), as a consequence of which they are not presented here. Thus, from the analysis just carried out it is seen that for $\mathrm{Re}_m \ll 1$ the small parameter determining the asymptotic behavior of the quantities F_1, Q_1, η for large k_0 is furnished by the parameter

$$\varepsilon = (\sqrt{\mathrm{Re}_m k_0}\mathrm{Re}_m)^{-1}, \tag{5.5}$$

with

$$F_1 = F_\infty\left[1 + O\left(\varepsilon^2 \mathrm{Re}_m^2\right)\right], \quad Q_1 = Q_\infty[1 + O(\varepsilon)], \quad \eta = \eta_\infty[1 - O(\varepsilon)]$$

(the parameter 1 – s here is considered to be finite).

It is necessary to make the following remark. Factually, when approaching the condition (5.4), the given analysis ceases to be valid, since in this case the electromagnetic fields will be concentrated within the boundary layer and the assumption (1.4) is violated. But one thing is clear – a very large number of waves is necessary, to be able to go to the "infinite plate" regime giving the efficiency η = s. Consequently, in this case it is not possible to use the results obtained from the analysis of an infinitely long periodic source, for a body of finite dimensions, as is done in [2].

6. The question about the possibility of controlling the energy quantities F_1, η by means of "amplitude modulation" is of interest. The term "amplitude modulation" here is used to denote the fact that the distribution of amplitude of current (1.1) across the width of the plate is different from a uniform distribution, i.e., $i_0(x) \not\equiv 1$.

Above it is noted that the behavior of the functions $F_1(k_0)$, $Q_1(k_0)$ in the case of fixed Re_m, s depends on the function. It is understood that if we take $|I(k-k_0)|^2$ such that the decay of the secondary maxima of the function $i_0(x)$ is more rapid than for $|I(k-k_0)|^2$, then the contribution of the neighborhood of the point k = 0 to the integrals is reduced. Since this contribution is especially large for the quantity Q_1 (because of the large value of the multiplier Φ(k) in the integral Q) the circumstance just mentioned should lead to an increase in η.

We consider the example

$$i_0(x) = \cos \pi x \ (|x| \leqslant 1/2), \quad I(k) = \cos (k/2)/(\pi^2 - k^2), \tag{6.1}$$

for which the law of decay of the secondary maxima $|I(k)|^2$ for large k is $1/k^4$. At the same time, if $1/k_0^4 \ll \mathrm{Re}_m(1-s)/k_0$, the neighborhood of the point k = 0 ceases to influence F_1 and the relation $F_1(k_0)$ assumes the asymptotic value $(1/4\pi)\mathrm{Re}_m(1-s)/2k_0$, differing by the multiplier 1/2 from the expression F_∞ (5.1), i.e.,

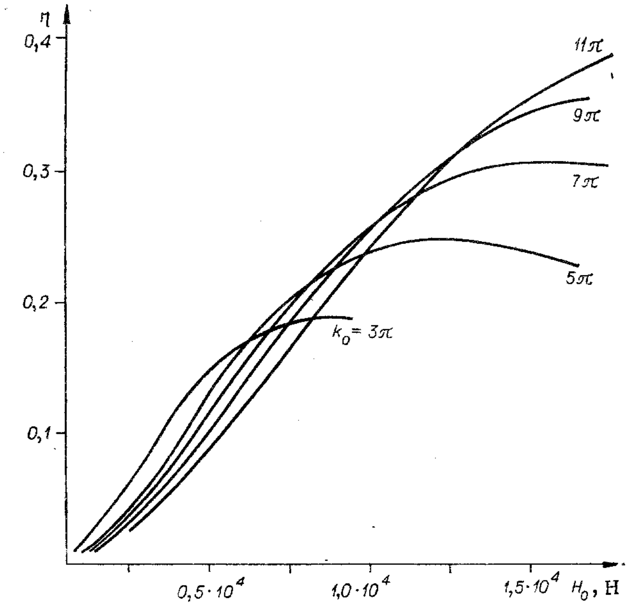

Fig. 6

$$F_1 = (\mathrm{Re}_m (1 - s)/8\pi k_0) \left[1 + O\left(\varepsilon^3 \mathrm{Re}_m^2\right)\right],$$

where ϵ is determined according to (5.5). For $(\sqrt{2\mathrm{Re}_m k_0 k_0^4})^{-1} \ll \mathrm{Re}_m(1-s)/2k_0^2 Q_1$ ceases to depend on the neighborhood $k = 0$ and the quantity goes over to the asymptotic value of s, i.e.,

$$\eta = s\,[1 - O(\varepsilon^{5/2}\mathrm{Re}_m)].$$

The results of computations for the example (6.1) under consideration are presented in Fig. 5 (for the value considered above $\mathrm{Re}_m^0 = 0.02$). Here solid lines depict F_1, and dashed lines depict η. We see that the oscillatory character of the relation $F_1(k_0)$ already for $k_0 > 3\pi$ manifests itself slightly. As for η, then this quantity, although it preserves an oscillatory character within the limits $k_0 \leqslant 15\pi$, has a clear tendency towards growth when k_0 increases. For $s = 0.2$ η approaches the asymptotic value 0.2 already for $k_0 = 15\pi$; for larger s the asymptotic values of s are attained, as is seen from Fig. 5, for $k_0 > 15\pi$. (For the value $s = 0.8$ the function $\eta(k_0)$ is not plotted on the graph, since the maxima of η for the given s and $k_0 < 15\pi$ lie below the maxima of η corresponding to $s = 0.6$). It is necessary to emphasize that use of "amplitude modulation," as comparison of the results of Figs. 2 and 5 shows, allows us considerably to increase the value η for the given values of the parameter k_0. Precisely the fact that relatively large values of η are attained for small k_0, for which the dimensionless force F_1 still is not very small and, consequently, the required quantities H_0 can be considered as practicable.

In Fig. 6 we have presented the dependence of efficiency on the magnitude of H_0 for fixed values of k_0 ensuring local maxima of $\eta(k_0)$. Here along the abscissa axis we have plotted the values H_0 calculated according to the expression (4.2) for the dimension $\mathrm{Re}_m^0 = 0.02$. $2a = 200$ m.

We see that if on the plate there are packed three half-waves of current, i.e., $k_0 = 3\pi$, then independently of H_0 the efficiency cannot be higher than 19%; for $k_0 = 5\pi\eta$ it does not exceed 25%; to each permissible value of H_0 there corresponds its value of k_0 ensuring the maximum efficiency; an increase in the permissible H_0 necessitates going over to a larger number of half-waves and ensures higher values of η. In Fig. 6 the maximum value of k_0 is taken as 11π; the values 13π, 15π and above reveal that η is higher for values of H_0 going beyond the limits used in Fig. 6; therefore the corresponding curves are not plotted here (to the maximum value taken here $H_0 = 1.75 \cdot 10^4$ G there corresponds the value of parameter N = 0.24). For the example we can note that for $k_0 = 15\pi$, as is seen from Fig. 5, the value $\eta \simeq 45\%$ is attained; here $H_0 \simeq 22 \cdot 10^3$ G.

Here the advantages of use of "amplitude modulation" are shown on the example (6.1). It is obvious that if we take an "impulse" yet with a narrower spectrum (e.g., the Gaussian curve $e^{-\alpha x^2}$ possesses a minimum spectral width [9]), then these advantages must show up still more clearly.

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SELF-SIMILAR MOTION OF AN IONIZED GAS EXPELLED BY A MAGNETIC PISTON

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The motion of a gas in plasma accelerators and high-current discharges, under the conditions of the skin effect, can be represented as its ejection by a magnetic piston under the action of a given current flow along the surface. Such a model was first proposed in [1] to explain the pinch effect. In the initial stage, the law of the rise in the current is approximated rather well by a linear function of the time, and the magnetic field, by a quadratic law: $p = Ct^n$, where $n = 2$; $C = \text{const}$. Under the usual conditions of an experiment, the magnetic pressure is much greater than the initial pressure of the gas, and the latter can be neglected. In this case, the motion of the gas is self-similar. An analogous problem for a given law of change in the velocity of the piston was discussed earlier [2, 3].

We shall assume the gas to be ideal and monoatomic, and the process to be adiabatic. The determining parameters in the problem will be the coordinate r, the time t, the density of the unperturbed gas ρ_1, and the constant C, determining the law of change in the pressure at the piston (the initial velocity $v_1 = 0$ and the initial pressure $p_1 = 0$). From these parameters, a single dimensionless variable can be obtained

$$\lambda = \sqrt{\frac{C}{\rho_1}}\frac{t^{n/2+1}}{r} = \sqrt{\frac{C}{\rho_1}}\frac{t^{m+1}}{r}, \quad m = \frac{n}{2}.$$

For the velocity, the density, and the pressure, we introduce the dimensionless functions V, R, P in the following manner:

$$v = \frac{r}{t}V(\lambda), \quad \rho = \rho_1 R(\lambda), \quad p = \frac{\rho_1 r^2}{t^2}P(\lambda), \quad z = \frac{\gamma P}{R},$$

then, the system of hydrodynamic equations is brought into the form [4]

$$\frac{dz}{dV} = z\frac{[2(V-1)+\nu(\gamma-1)V](V-m-1)-(\gamma-1)V(V-1)(V-m-1)-\left[2(V-1)-2m\frac{\gamma-1}{\gamma}\right]z}{(V-m-1)[V(V-1)(V-m-1)-(2m/\gamma+\nu V)z]}; \tag{1}$$

$$\frac{d\ln\lambda}{dV} = \frac{(V-m-1)^2-z}{V(V-1)(V-m-1)-(2m/\gamma+\nu V)z}; \tag{2}$$

$$\frac{d\ln R}{d\ln\lambda}(V-m-1) = \frac{V(V-1)(V-m-1)-(2m/\gamma+\nu V)z}{z-(V-m-1)^2}+\nu V, \tag{3}$$

where γ is the ratio of the specific heat capacities; $\nu = 1, 2, 3$, respectively, for plane, cylindrical, and spherical symmetry.

Let us examine the additional conditions which arise due to the presence of the surface of a strong discontinuity ahead of the piston. We note that, in the shock wave, r is a function of t. Consequently, the determining parameters in the shock wave will be t, ρ_1, and C, from which it is impossible to form a dimensionless quantity. Therefore, at the shock wave

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